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COMMENT

Comment on 'On the Kowalevsky–Goryachev–Chaplygin gyrostat'*

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Abstract
Comment on the letter by A V Tsiganov 2002 *J. Phys. A: Math. Gen.* **35** L309.

The commented letter to the editor, which we shall refer to as L, deals with the separation of variables in an integrable case of motion of a rigid body about a fixed point under the action of certain potential forces and a constant gyrostatic moment. The inertia matrix of the body is assumed to have the Kovalevskaya configuration $I = \mathrm{diag}\left(1, 1, \frac{1}{2}\right)$. The problem is characterized by the Hamiltonian L(1.1)

$$H = J_1^2 + J_2^2 + 2J_3^2 + 2\rho J_3 + c_1 x_1 + c_2 x_2 + c_3\left(x_1^2 - x_2^2\right) + c_4 x_1 x_2 + \frac{\delta}{x_3^2} \tag{1}$$

and it is integrable on the zero level of the cyclic integral of motion

$$\sum_{i=1}^{3} J_i x_i = 0. \tag{2}$$

It is evident that integrable cases constitute a rare exception in Hamiltonian dynamics in general. The field of rigid body dynamics reflects this fact even more clearly than any other field. To demonstrate this rarity, it may be sufficient to recall that in the more general problem of motion of a rigid body under the action of forces with potential $V(x_1, x_2, x_3)$ and gyroscopic moment $\boldsymbol{\mu}(x_1, x_2, x_3) \times \boldsymbol{\omega}$ ($\boldsymbol{\omega}$ being the angular velocity), there are only sixteen integrable cases known up to the present time. Fifteen cases were tabulated in our work [1] and a brief history is pointed out for each case. Seven of those cases are general and eight are restricted by the condition (2). One conditional case is added in [2].

Unfortunately, the above-mentioned letter to the editor has brought some confusion concerning literature in this field. We have the following remarks:

(a) The title of L attributes this integrable case to Kovalevskaya, Goryachev and Chaplygin. For the Hamiltonian L(1.1), the author gives the reference L [1] including one work of Kovalevskaya [3], two works of Chaplygin [4] and one of Goryachev [5].

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1. Kovalevskaya's case ($\rho = c_3 = c_4 = \delta = 0$) is a general case while the case of L is conditional.
2. Chaplygin's two cited papers are not at all related to the case cited in L. The proper reference should be [7] (L [2]), in which Chaplygin pointed out the fourth integral for the case ($\rho = \delta = 0$) and gave the explicit time solution of the case ($\rho = c_1 = c_2 = \delta = 0$).
3. Goryachev's paper in the citation L [1] also deals with a classical, but completely different, conditional integrable case known in the literature as 'Goryachev–Chaplygin's case' and valid only when the moments of inertia satisfy the condition $A = B = 4C$. This condition figures in the title of the original paper [5] but it was dropped from the citation title in L [1]. The proper reference should be another work [6] of the same author, in which he gave the integrable case involving the constants $c_1, c_2, c_3, c_4, \delta$, but without the gyrostatic term ($\rho = 0$). This work generalizing Chaplygin's result for the first time by the presence of the singular term $\frac{\delta}{x_3^2}$ is not cited in L.
4. None of the four works in L [1] dealt with the motion of the gyrostat. None can be a source for the Hamiltonian L(1.1) attributed to them all. The Letter's title is also misleading, since the three authors have no results concerning the motion of a gyrostat.

(b) The additional integral K L(1.6) is given without any reference. The first case under the Kovalevskaya configuration involving the gyrostatic parameter ρ was introduced in our work [8] and also included in [9, 10]. One year later, the same case was obtained independently by Komarov [11] and by Gavrilov [12]. This was a general integrable case generalizing the full case of Kovalevskaya. The gyrostatic moment was added to Chaplygin's case without Goryachev's singular term in [13]. In [14], this term was added and the additional integral obtained for the full case covering L(1.1). None of these works is cited in the commented letter.

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